DETERMINATION OF CRITICAL VELOCITY OF SUSPENSION FLOW

S. E. Saks

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Suspension of solid particles in air flow is considered and flow critical velocity is determined. A comparison is made of computational and experimental results.

When designing engineering or transportation systems with suspension flows it is essential to have a correct determination of the critical velocity of transportation. In the literature, by critical velocity (obstruction velocity) in horizontal tubes one usually understands the velocity of aeromixture flow at which particle precipitation commences. Sometimes one comes across a different definition of critical velocity, namely, defined as the velocity at which buoyancy of the particles at the bottom starts. It will be shown below that both definitions correspond to different limiting cases of the process of particle suspension in a flow. The latter explains why different experimental values of critical velocities are obtained by various investigators though the properties of the transported materials are very similar.

In engineering plants the bandwidth of the values of Re for pneumatic transportation is usually $5 \cdot 10^4$-$5 \cdot 10^5$. Then the flow motion becomes turbulent and the effect of both the averaged and the pulsating motions of the carrier agent is checked by the solid particles. The motion of particles due to turbulent pulsations of the agent was analyzed in [1, 2]. The results show that with the particles' size or density increasing, the effect of turbulent pulsations of the carrier agent on their motion is reduced [2]; for particles whose size exceeds 0.1-0.2 mm the turbulent transfer in air flow becomes insignificant. In this case the mechanism of particle buoyancy is related to the effect of the field of full instantaneous flow velocities, the order of the quantity of the buoyancy forces being determined by the value of the averaged velocity field at a given flow point; the effect of turbulent velocity pulsations is a kind of random excitation superimposed on the basic relationship.

A stationary aeromixture flow is now considered which takes place in a straight-line horizontal tube of constant cross-section; the flow is in the direction of the x-axis which is identical with the bottom of the tube ($y = 0$). Moreover, it is also assumed that on the selected portion the pressure differential is small and that the air compressibility can be ignored (ρ_0 = const).

It follows from the assumptions made above that U_{ep} = const and $dU_x/dx = 0$. Moreover, one has $U_y = 0$ for a stationary one-dimensional flow in a tube. Since there are no averaged vertical velocities of the carrier agent and the pulsation velocities of the particles are small (for big particles the pulsation velocities are much smaller than the turning velocities) the appearance of a buoyant force in such flows can only be explained by the inhomogeneity of the velocity field in a cross-section.

The relation between the buoyant force and the velocity gradient was analyzed in [3, 4]; in [3], however, a linear velocity profile was assumed in the analysis which hardly corresponds to an actual flow profile; in [4] only a special case was considered of a power profile with $n = 1/7$. Moreover, in both articles the effect was considered of only the averaged velocity field and the stochastic character of the turbulent flow had not been taken into account.

A more general case will now be considered by us; it is assumed in the first approximation that velocity turbulent pulsations can be ignored.

In a planar circulatory flow with a velocity gradient of a cylindrical particle there arises a force which is perpendicular to the flow direction (the Magnus effect):

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$$F_y = -\rho_0 |\bar{U}_x| \Gamma, \quad (1)$$

where

$$\Gamma = \iint\limits_{\omega} \left(\frac{\partial \bar{U}_y}{\partial x} - \frac{\partial \bar{U}_x}{\partial y} \right) d\omega.$$

Similarly as in the case of pneumatic transportation, the particle dimensions are in most cases considerably smaller than those of the flow; therefore, at a distance d it can be assumed that $\partial \bar{U}_x / \partial y \approx$ const. Then

$$\Gamma = -\frac{d\bar{U}_x}{dy} \omega.$$

For a particle of length l one has $F_y \sim l$. Consequently,

$$F_y = \rho_0 \bar{U}_x \frac{d\bar{U}_x}{dy} \omega l. \quad (2)$$

Formula (2) determines the force acting on a fixed particle in a flow. Applying (2) in the case of a particle at the bottom of the tube one is able to determine the region of stable transportation. Such an approach is related to a simplified schematization of the problem since the form of a particle is never cylindrical; also the flow past a particle in a region near the bottom is different from the flow of unlimited size. However, it will be shown below that such a schematized approach enables one to obtain computational formulas which agree satisfactorily with the experimental data for a wide range of tube diameters and sizes of particles.

Stable transportation can take place if $F_y > G$. In the case of $F_y < G$ there is no buoyancy of particles and there is no transportation. The case $F_y = G$ describes the limiting particle equilibrium separating the transportation and nontransportation of particles; the corresponding flow velocity can be regarded as critical.

We shall use the notation: $U_{ave} = V$ and $\bar{U}_x = v$ for $F_y = G$.

The distribution of the averaged flow velocities is given by the formula

$$\bar{U}_x = U_{max} \left(\frac{y}{R} \right)^n, \quad (3)$$

where in the general case $n = f(Re)$.

It follows from the formula (3) that

$$\frac{d\bar{U}_x}{dy} = \frac{2n}{D} \bar{U}_x \left(\frac{y}{R} \right)^{-1}.$$

Then

$$F_y = \frac{2n}{D} \rho_0 (\bar{U}_x)^2 \left(\frac{y}{R} \right)^{-1} \omega l.$$

For a particle in the region near the bottom one has $y = d/2$ and $y/R = d/D$. Since $G = mg = \rho_s g \omega l$ then

$$\frac{F_y}{G} = \frac{2n (\bar{U}_x)^2}{agD} \left(\frac{d}{D} \right)^{-1},$$

where $a = \rho_s / \rho_0$.

By setting $F_y/G = 1$ and $\bar{U}_x = v$ one obtains

$$v = \sqrt{\frac{agD}{2n} \frac{d}{D}}. \quad (4)$$

The U_{max} velocity can be found from U_{ave} [5]:

$$U_{max} = \alpha_1 U_{ave}, \quad (5)$$

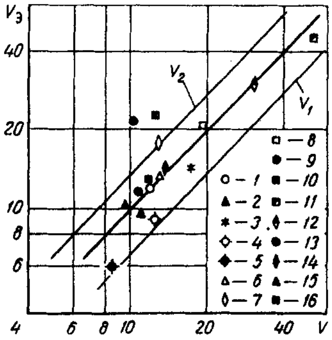

Fig. 1. Comparison of experimental and computed values of critical velocity (m/sec): 1) wheat, d_{ave} = 2.59 mm; 2) broken grain, d_{ave} = 2.08 mm; 3) fine gravel, d_{ave} = 2.4 mm; 4) sand, d_{ave} = 0.9 mm; 5) sand, d_{ave} = 0.342 mm, D = 61 mm (Welshoff); 6) wheat, d_{ave} = 3 mm, D = 113 mm (Zegler); 7) wheat, d_{ave} = 1.9 mm; 8) peas, d_{ave} = 5.76 mm; 9) sand, d_{ave} = 0.42 mm; 10) sand, d_{ave} = 0.715 mm; D = 125 mm (Dogin and Lebedev); 11) rock, d_{ave} = 25-30 mm, D = 150 mm (Mikhailov and Smoldyrev); 12) coal, 0-13 mm, D = 200 mm (the author and Kozhushko); 13) coal, 0-1.25 mm; 14) coal, 1.25-2.5 mm; 15) sand, 0.14-0.315 mm; 16) sand, 0.315-0.63 mm, D = 106 mm (the author's experiments).

where

$$\alpha_1 = \frac{(n+1)(n+2)}{2}.$$

It follows from (3) and (5) that

$$U_{ave} = \frac{\overline{U}_x}{\alpha_1}\left(\frac{d}{D}\right)^{-n}.$$

Employing (4) one finds the flow-averaged value of the critical velocity,

$$V = \frac{v}{\alpha_1}\left(\frac{d}{D}\right)^{-n} = \alpha_2\sqrt{agD\left(\frac{d}{D}\right)^{1-2n}}, \tag{6}$$

where

$$\alpha_2 = \frac{\sqrt{2}}{\sqrt{n}(n+1)(n+2)}.$$

For the widest range of Re numbers encountered in practice one has:

for $5\cdot10^4 < \mathrm{Re} \le 10^5$, n = 1/7

$$V = 1.53\sqrt{agD\left(\frac{d}{D}\right)^{\frac{5}{7}}}, \tag{7}$$

for $10^5 < \mathrm{Re} \le 5\cdot10^5$, n = 1/8

$$V = 1.67\sqrt{agD\left(\frac{d}{D}\right)^{\frac{3}{4}}} \tag{8}$$

To determine the critical velocities for a variety of materials experimental investigations have been carried out in two setups. In a laboratory with a horizontal tube of D = 106 mm the investigations were carried out of the transportation of narrow fractions of coal (0-1.25 mm and 1.25-2.5 mm) and sand (0.14-0.315 mm and 0.315-0.63 mm). In a plant on a pneumatic-transport layout with a tube of D = 200 mm and the length of the horizontal route of 250 m the experiments were carried out with run-of-mine coal of size 0-13 mm.

The experimental values of critical velocity V_{exp} (using the data of [6]-[9] and the average experimental results obtained by the author) are compared in Fig. 1 with the corresponding computed values of V using the formulas (7) and (8). It can be seen from the data that there is a sufficiently good agreement between the computed values V and the average values of critical velocities obtained from the data of various scientific workers though there is a smaller or greater amount of deviation in individual experimental points. The latter is due to the stochastic character of the turbulent flow.

Since the formulas (7) and (8) were obtained only from a theoretical analysis and do not contain any empirical coefficients, it can be concluded that the satisfactory agreement of the computed and experimental data confirms the correctness of the initial theoretical model of a solid particle moving in flow.

A shortcoming of the derived formulas lies in that the effect of the aeromixture concentration is ignored. For high concentrations there is a change in the velocity distribution in a flow cross-section as well as in the velocity gradient in the region near the bottom. It would thus be expedient to introduce in the formulas (7) and (8) a coefficient so that these changes are taken into account.

Moreover, since the computation results are very close irrespective of whether the formula (7) or (8) is used, it is advisable to introduce only one unified formula suitable to be employed in engineering calculations. By using the average value of the theoretical coefficient α_2 one obtains

$$V = 1.6\alpha_{conc} \sqrt{agD \left(\frac{d}{D}\right)^{\frac{3}{4}}}, \tag{9}$$

where α_{conc} is an experimental coefficient with the effect of concentration taken into account. It is recommended that $\alpha_{conc} = 1$ for $\mu \leq 5$ kg/kg and $\alpha_{conc} = 1.1$ for $5 < \mu \leq 10$ kg/kg.

The motions of aeromixture and pure air are turbulent. It would thus appear that a statistical approach based on probability of suspension of a particle would be a very general and natural approach. Therefore, we shall now consider as another approximation not the effect of the averaged flow velocities but of the actual instantaneous ones U_x; thus, at a given flow point U_x is regarded as a random time function. If $\bar{U}_x > u$ then the individual instantaneous values of velocity may prove less than critical and there is no buoyancy of the particle. Conversely, if $\bar{U}_x > v$ then there exists a nonvanishing probability of particle buoyancy at different instants.

Therefore from the statistical point of view it is proper that one does not speak of critical velocity but of critical velocity interval. This critical interval incorporates the region $v_1 < U_x < v_2$ in which the particle buoyancy probability is $0 < P_0 < 1$. The above given two formulas for the critical velocity correspond to both ends of the critical interval, v_2 and v_1.

The statistical approach to particle suspension applicable to near-bottom drifts was used in the Einstein–Velikanov theory [10]. However, a similar method is employed by us in a more suitable form. Following Velikanov, it is assumed that the instantaneous flow velocities are normally distributed. Then the probability of a particle break-away in terms of the integral distribution function is as follows:

$$P_0 = P(F_y > G) = P(U_x > v) = 1 - P(U_x < v) = 1 - F(v),$$

or

$$P_0 = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\beta} e^{-\frac{z^2}{2}} dz, \tag{10}$$

where

$$\beta = \frac{v - U_x}{\sigma_x}.$$

The calculations using the formula (10) show that with an error up to 1% for $v_1 = v - 2.3\,\sigma_x$ one has $P_0 = 0$ and for $v_2 = v + 2.3\,\sigma_x$ one has $P_0 = 1$.

Experimental investigations of airflow in tubes [11] show that in the wall-adjacent region at the height of about 0.01 D one has $\sigma_x = 0.15\,\bar{U}_x$. Consequently, $v_1 = 0.65\,v$ and $v_2 = 1.35\,v$ and by using the formula (6) the limits are obtained of the critical range for the average flow velocity,

$$V_1 = 0.65V \text{ and } V_2 = 1.35V.$$

Both bounds V_1 and V_2 of the critical range are shown in Figure 1. It can be seen from the graph that all experimental results are within the critical band $V_1 < U_{ave} < V_2$. The middle straight line $V = V_{exp}$ corresponds to $P_0 = 0.5$ and is the central axis of the set of experimental points.

Thus the motion of an aeromixture flow for velocities close to the critical interval has the following special properties: for $U_{ave} < V_1$ one has $P_0 = 0$ and the particles are not blown away from the bottom; for $U_{ave} > V_2$ one has $P_0 = 1$ and all particles are suspended; for $V_1 < U_{ave} < V_2$ one has $0 < P_0 < 1$ and the motion is unstable; a periodic deposition of the transported material takes place and the pressure pulsations are considerable.

To ensure that the operation of a pneumatic transport system is stable, the operational transportation velocity should exceed the upper bound of the critical interval.

NOTATION

U_{ave} is the mean flow velocity;
U_{max} is the velocity at flow axis;
U_x, U_y are the projections of instantaneous velocity at an arbitrary flow point;
$\bar{U}_x$, $\bar{U}_y$ are the projections for the average velocity;
V is the critical velocity (average over section);
v is the critical velocity (at flow point, also $\bar{U}_x = v$ for $U_{ave} = V$);
σ is the mean-square value of turbulent velocity pulsations;
Γ is the velocity circulation;
n is the power index in formula for velocity profile;
F_y is the buoyancy force on particle;
D is the tube diameter;
R is the radius of tube;
d is the particle diameter;
ω is the area of particle section;
m is the particle mass;
G is the particle weight;
l is the particle length;
g is the gravitational acceleration;
ρ_0, ρ_s are the density of air and of solid particles respectively;
P is the probability;
P_0 is the particle separation probability;
F is the integral distribution function;
μ is the weight concentration of aeromixture.

LITERATURE CITED

1. S. E. Saks, IFZh, 9, No. 4 (1965).
2. S. E. Saks, IFZh, 14, No. 4 (1968).
3. V. A. Shvab, Trudy TÉMIIT, 23 (1957).
4. M. I. Solov'ev, IFZh, 7, No. 10 (1964).
5. L. G. Loitsyanskii, Mechanics of Fluids and Gases, Fizmatgiz, Moscow (1959).
6. G. Welshoff, Pneumatic Transport for High Concentrations of the Moved Material, Kolos, Moscow (1964).
7. G. Zegler, Pneumatic Grain Transportation, Kharkov (1937).
8. P. G. Mikhailov and A. E. Smoldyrev, Nauchnye Soobshcheniya IGD AN SSSR, No. 3 (1959).
9. M. E. Dogin and V. P. Lebedev, Trudy TÉMIIT, 29 (1960).
10. M. A. Velikanov, Dynamics of River-Bed Flows, Vol. 2, Gostekhteorizdat, Moscow (1956).
11. J. Laufer, Natl. Advisory Comm. Aeronaut. Tech. Repts., No. 1174 (1954).